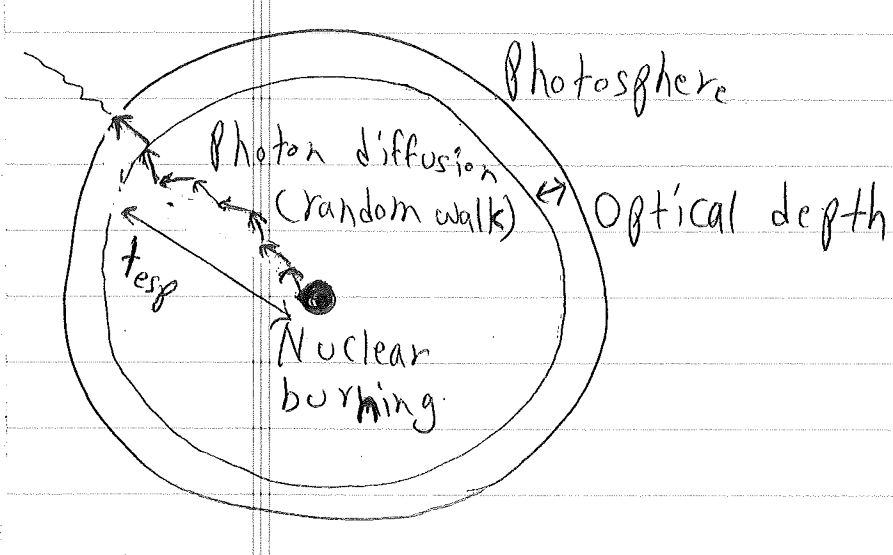
Lec 4: 09/02/2010

Stellar Magnitude and Colors:

Transport of energy in the stars happens via different means. Lets focus on transport from central regions (where nuclear burning takes place) to outer regions via photon diffusion. The opacity of matter plays a vital role here; particularly determining the relation between the luminosity and the mass of the star.

Schematically, we have the following picture for photon diffusion:



Photons diffuse in random walk fashion. The mean free path of photons is given by:

$\ell = \frac{1}{n\sigma}$ (n: number density of targets, σ: cross section)

Being a random walk, the number of collisions occuring until a photon reaches the surface is:

$N_{col} \approx \left(\frac{R}{\ell}\right)^2$ (R: radius of the star)

The escape time is then given by:

$$t_{esc} \approx \frac{\ell N_{col}}{c} \approx \frac{R^2}{\ell c}$$

The radiant energy content of the star (generated by nuclear burning) is $E_\gamma \propto T^4 R^3$ (energy density in photons at thermal equilibrium is $\propto T^4$).

The luminosity (energy flux per unit time) will be:

$$L \sim \frac{R^3 T^4}{t_{esc}} \propto \boxed{\frac{R^3 T^4 \ell}{R^2} \propto \frac{R T^4}{n\sigma}} \quad *$$

For a wide class of stars $k_B T \sim \frac{G M m_p}{R}$ (M being the mass of the star). This is what we found as a condition for hydrostatic equilibrium of a non-degenerate gas. It holds as long as T remains constant in the presence of nuclear reactions that act as a thermostat.

The cross section σ depends on interactions of the photon with the matter. We consider two important interactions:

1) Thomson scattering. If scattering of photons off free electron dominates, then we will have:

$$\sigma = \sigma_T \equiv \frac{8\pi}{3} \frac{e^2}{m_e c^2} \simeq 0.67 \times 10^{-24} \, cm^2 \quad (\tfrac{2}{3} \text{ barn}, 1 \text{ barn} \equiv 10^{-24} \, cm^2)$$

In this case, we find from equation $*$:

$$L \propto \frac{RT^4}{\sigma_T n} \propto \frac{T^4 R^4}{\sigma_T N} \propto \boxed{\frac{M^4}{M} \propto M^3} \qquad **$$

2) Photoionization. The situation will be different if interaction of photons with partially ionized atoms provide the opacity. In this case we have:

$$l \propto T^{\frac{7}{2}} n^{-2} \propto T^{\frac{7}{2}} R^6 M^{-2}$$

This implies:

$$L \propto RT^4 \ell \propto R^7 T^{\frac{15}{2}} M^{-2} \propto M^{\frac{11}{2}} R^{-\frac{1}{2}}$$

Using $k_B T \sim \frac{GMm_p}{R}$, and at constant temperature, we then find $M \propto R$. Thus:

$\boxed{L \propto M^5}$ ***

From equations *, **, *** we expect the luminosity of a star be related to its mass by $L \propto M^\alpha$, wher $\alpha \simeq 3-5$. If we imagine the surface of the star to be at some effective temperature T_s, then the total blackbody luminosity will be:

$L = 4\pi R^2 (\sigma T_s^4)$ (σ: Stefan-Boltzmann Constant)

The intensity f_ν (energy per unit area per unit time per solid angle per frequency) of thermal radiation emitted by the star will be:

$$\boxed{f_\nu \doteq \frac{dE}{dA\,dt\,d\Omega\,d\nu} = B_\nu \equiv \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/k_BT_s}-1}}$$

The quantity νB_ν reaches a maximum at $h\nu \simeq 4k_B T_s$.

For Sun ($T_s \simeq 6000$ K) the maximum happens at $\lambda \simeq 6000$ Å.

The above description provides useful observed characteristics of a star. One can:

- find T_s by fitting the spectrum of a star to a blackbody spectrum.
- measure the intrinsic luminosity if the distance to the star is known (recall that $I = \frac{L}{4\pi d^2}$, "d" being the distance).
- Plot the location of a star in the $L - T_s$ plane (assuming that the distance can be measured independently). If the stars plotted on the $\log L - \log T_s$ plane, we will have the H-R (Hertzsprung-Russel) diagram.

As we saw $L \propto M^{\alpha}$, where $\alpha \simeq 3-5$. Using $T_s \propto L^{\frac{1}{4}} R^{-\frac{1}{2}}$

and $M \propto R$, we findfrom equations **, **;

$$\boxed{\left\{\begin{array}{ll} T_s \propto L^{\frac{1}{4}} L^{-\frac{1}{10}} \propto L^{3/20} & \text{Photoionization dominates} \\ T_s \propto L^{\frac{1}{4}} L^{-\frac{1}{6}} \propto L^{\frac{1}{12}} & \text{Thomson scattering dominates} \end{array}\right.}$$

Hence we expect the stars to lie within the lines with slopes $\frac{3}{20} = 0.15$ and $\frac{1}{12} \simeq 0.08$ on the $\log T_s - \log L$ plane.

The observed slope is $\underline{\underline{0.13}}$.

Observationally, the following ranges for M, T_s, R, L are found:

$M \simeq (0.1 - 60)\, M_{\odot}$

$T_s \simeq (3\times 10^3 - 3\times 10^4)\, K$

$R \simeq (0.8 - 70) \times 10^{10}$ cm

$L \simeq (10^{-3} - 10^{5.7})\, L_{\odot}$